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The revealing properties of a rational expectations equilibrium - an extension of Radner's auxiliary proposition

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Abstract

In this note we extend Radner's ([6]) result on the revealing properties of a rational expectations equilibrium to the case of an infinite dimensional probability space. Radner's *auxiliary proposition*, which states that the set of probability assessments leading to the same equilibrium price is negligible, is generalised to the infinite dimensional case. In the original paper a set is negligible if its closure has Lebesgue measure zero in $\mathbb{R}^N$, while in our setting a set is negligible if it is a meagre subset of some topological space.

Introduction

In [6], Radner studies the revealing properties of a rational expectations equilibrium in a fairly general model of a two-period market model in a finite dimensional probability space. His *auxiliary proposition* states that situations where equilibrium prices fail to reveal the agents' information are "rare" in the sense that the set of different probability assessments leading to the same equilibrium price is *negligible*. A property that holds everywhere except on a negligible set is said to hold *generically*. Hence we can say that generically, different probability assessments lead to different equilibrium prices. In Radner's finite dimensional setting a set is negligible if its closure has zero Lebesgue measure. Though the Lebesgue measure makes no sense in an infinite dimensional space, the concept of genericity is well-defined for a much wider class of topological spaces: A property is said to hold generically in a *Baire space* if it holds everywhere except on a *meagre* subset, i.e. a set that can be expressed as a countable union of sets that are nowhere dense.

Radner's result has been generalised and extended in a variety of directions, see e.g. [3] for an overview. The infinite dimensional case is studied in [1] in a different setting from the present. In this note we remain quite true to Radner's original setting, with the following simplifications:

- We do not allow *heterogeneous beliefs*. Our agents' probability assessments are given uniquely by their information or signal[1].

- In our market model, short selling is allowed, cf. Remark 4.2.

The note is organised as follows: Section 1 introduces the probabilistic setting and the financial market. Radner's different equilibrium concepts are modified to fit our present framework in Section 2. Section 3 deals with spaces of conditional probability measures and the topological properties of genericity and meagreness. The auxiliary proposition is stated and proved in Section 4.

1 The agents and the assets

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space and assume that $\mathcal{F}$ is separable. The J stocks are traded at time 0 and has the $\mathcal{F}$-measurable $\mathbb{R}^J$-valued time 1 payoff V. We let $\mathcal{F}_V$ denote the σ-algebra generated by V and assume that $P(F) > 0$ for all non-empty $F \in \mathcal{F}_V$. There are I agents in the market. Agent i receives an initial endowment $\epsilon^{(i)} \in \mathbb{R}_+$ of cash and $e^{(i)} \in \mathbb{R}_+^J$ of stocks. The agents' utility functions are of the form

$$U_{0i}(\text{time 0 consumption}) + U_i(\text{time 1 wealth}).$$

The agents' time 0 decisions are based on their initial information given by the σ-algebra[2] $\mathcal{G} \subseteq \mathcal{F}$. Given an $\mathbb{R}^J$-valued $\mathcal{G}$-measurable asset price vector ϕ, agent i's choice of initial consumption c and portfolio of stocks z must be $\mathbb{R}$- and $\mathbb{R}^J$-valued $\mathcal{G}$-measurable random variables satisfying the *budget constraint*

$$c + \phi^\top z \leq \epsilon^{(i)} + \phi^\top e^{(i)} \qquad \text{a.s.} \tag{1.1}$$

We will work under assumptions ensuring that the budget constraints hold with equality and that the solution to the optimisation problem

$$\max_{z \in \mathbb{R}^J} \left\{ E\Big[U_{0i}\big(\epsilon^{(i)} + \phi^\top (e^{(i)} - z)\big) + U_i(V^\top z) \big| \mathcal{G} \Big] \right\} \tag{1.2}$$

exists and is unique. Hence, given a $\mathcal{G}$-measurable ϕ, $z^{(i)}(\phi) : \Omega \to \mathbb{R}^J$ solving (1.2) is a $\mathcal{G}$-measurable $\mathbb{R}^J$-valued random variable. Throughout the text, the collection $z^{(1)}, \ldots z^{(I)}$ of agents' demands will be denoted by the shorthand $(z^{(i)})$.

[1]For readers familiar with Radner's paper, this amounts to studying P^2 in stead of P^{2I}. For more on information vs. signal see Note 2

[2]Economists tend to prefer the term *signal* to describe an agent's information - in our setting $\mathcal{G}$ can thus be thought of as the σ-algebra generated by some signal function.

2 Communicating equilibria and rational expectations equilibria

Definition 2.1 ((revealing) full communication equilibrium)**.** A *full communication equilibrium* is a collection $\{(z^{(i)}), \phi\}$ of $\mathcal{G}$-measurable $\mathbb{R}^J$-valued random variables such that for any i, $z^{(i)}$ solves (1.2) and the asset market clears, i.e.

$$\sum z^{(i)} = \sum e^{(i)} \qquad \text{a.s.} \tag{2.1}$$

A full communications equilibrium is *revealing* if

$$\sigma\{\phi\} \bigwedge \mathcal{F}_V = \mathcal{G} \bigwedge \mathcal{F}_V. \tag{2.2}$$

Note that (2.1) implies $\sum c^{(i)} = \sum \epsilon^{(i)}$ a.s. when the budget constraints hold with equality.

Consider now the case where the agents come to the market with *different* information $\mathcal{G}_1, \ldots, \mathcal{G}_I$ and denote the pooled information

$$\mathcal{G} := \mathcal{G}_1 \bigvee \ldots \bigvee \mathcal{G}_I.$$

We could of course proceed naively and define a "no communications equilibrium" as a collection $\{(z^{(i)}), \phi\}$ of $\mathcal{G}$-measurable $\mathbb{R}^J$-valued random variables such that $z^{(i)}$ solves (1.2) *with $\mathcal{G}$ replaced by $\mathcal{G}_i$* for each agent and (2.1) holds. But then we neglect the fact that *a sophisticated trader could use the equilibrium prices to extract information about the other agents' information.* This new information could in turn lead him to altering his demand for certain stocks. But if the total market demand changes significantly, the price vector is no more an equilibrium price vector. For a given $\mathbb{R}^J$-valued $\mathcal{G}$-measurable asset price vector ϕ, consider in stead the optimisation problem

$$\max_{z\in\mathbb{R}^J_+} \Big\{ E\Big[U_{0i}\big(\epsilon^{(i)} + \phi^\top (e^{(i)} - z)\big) + U_i(V^\top z) \big| \mathcal{G}_i \bigvee \sigma\{\phi\} \Big] \Big\}. \tag{2.3}$$

Definition 2.2 (rational expectations equilibrium)**.** A *rational expectations equilibrium* is a collection $\{(z^{(i)}), \phi\}$ of $\mathcal{G}$-measurable $\mathbb{R}^J$-valued random variables such that for any i, $z^{(i)}$ solves (2.3) and (2.1) holds.

Clearly, a revealing full communications equilibrium is also a rational expectations equilibrium.

3 Conditional probabilities

In the sequel, we shall deal with σ-algebras only indirectly via their conditional probabilities, or more precisely their conditional probabilities restricted to $\mathcal{F}_V$. Let $P(\cdot|\mathcal{G})$ denote a *regular version of the conditional probability* (cf. e.g. [4, Chapter VIII], [2, Section 33]). This measure is absolutely continuous with respect to P which implies that the restriction of $P(\cdot|\mathcal{G})$ to $\mathcal{F}_V$ is absolutely continuous with respect to the restriction $P|_{\mathcal{F}_V}$ on $(\Omega, \mathcal{F}_V)$. We let $\mathcal{P}$ denote the set of probability measures on $(\Omega, \mathcal{F}_V)$ that are absolutely continuous with respect to $P|_{\mathcal{F}_V}$. Fix some $q \in \mathbb{R}^J$, $\mu \in \mathcal{P}$ and consider

$$\max_{\zeta \in \mathbb{R}^J} \Big\{ U_{0i}\big(\epsilon^{(i)} + q^\top (e^{(i)} - \zeta)\big) + \int_\Omega U_i(V^\top \zeta) d\mu \Big\}. \tag{3.1}$$

We say that $q \in \mathbb{R}^J$ is an *equilibrium price* for μ if the collection $(\zeta^{(i)})$ of solutions to (3.1) satisfies

$$\sum \zeta^{(i)} = \sum e^{(i)}.$$

Definition 3.1 (confounding probability measures)**.** The measures μ, $\nu \in \mathcal{P}$ are *confounding* if they have a common equilibrium price.

Lemma 3.1. *The collection $\{(z^{(i)}), \phi\}$ of $\mathcal{G}$-measurable $\mathbb{R}^J$-valued random variables is a full communication equilibrium if and only if for any i and almost all ω, $z^{(i)}(\omega)$ solves* (3.1) *with $q = \phi(\omega)$ and $\mu = P|_{\mathcal{F}_V}(\cdot|\mathcal{G})(\omega)$, and* (2.1) *holds. Moreover, the equilibrium is revealing if the conditional probability measures corresponding to different sets in $\mathcal{G} \bigwedge \mathcal{F}_V$ are non-confounding.*

Proof. The first assertion follows from the fact that with the given q and μ, (3.1) is simply (1.2) pointwise. If the conditional probability measures corresponding to different sets in $\mathcal{G} \bigwedge \mathcal{F}_V$ are non-confounding, then

$$\sigma\{\phi\} \supseteq \mathcal{G} \bigwedge \mathcal{F}_V.$$

Hence, as ϕ is $\mathcal{G}$-measurable, (2.2) must hold. $\square$

Example 3.1 Suppose that $J = 2$, $\mathcal{F} = \sigma\{F_1, F_2, F_3\}$, $P(F_i) > 0$, $i = 1, \ldots, 3$ and

$$V(\omega) = \begin{cases} \begin{bmatrix} 2 & 1 \end{bmatrix}^\top & \omega \in F_1 \\ \begin{bmatrix} 1 & 2 \end{bmatrix}^\top & \omega \in F_2 \\ \begin{bmatrix} \frac{3}{2} & \frac{3}{2} \end{bmatrix}^\top & \omega \in F_3 \end{cases}$$

Suppose that each agent's utility functions and endowments coincide, i.e. $U_{0i} \equiv U_i \equiv U$ and $\epsilon^{(i)} \equiv \epsilon$, $e^{(i)} \equiv e$, given by

$$U(x) = 2\sqrt{x}, \qquad x > 0,$$
$$\epsilon = \frac{4}{3}, \; e = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top.$$

In this case $q = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$ is a (no-trade) equilibrium price for any probability measure μ with $\mu(F_1) = \mu(F_2)$. Hence any couple of probability measures assigning the same probability to F_1 and F_2 is confounding.

As Example 3.1 shows, one cannot in general rule out the occurence of confounding probability measures. It is possible, however, to show that the set of confounding measures is negligible. In the finite dimensional case studied in [6] with N denoting the number of states, $\mathcal{P}$ is the $N-1$-dimensional unit simplex[3] Δ_{N-1} equipped with the Lebesgue measure on $\mathbb{R}^{N-1}$ and the *auxiliary proposition* states that the set of confounding couples is negligible in the sense that its closure has zero Lebesgue measure in $\Delta_{N-1}^{\otimes 2}$. In a topological space a set is referred to as *meagre* if it can be expressed as a countable union of nowhere dense sets, i.e. sets for which the interior of the closure is empty. The complement of a meagre set is referred to as a *residual* set. A topological space is a *Baire space* if any residual set is dense. A property is said to hold *generically* in a Baire space if it holds on a residual subset. Any countable intersection of residual sets in a Baire space is in turn a residual set (cf. e.g. [5, Lemma 48.1]). Consequently, countable selections of generic properties hold simultaneously on a residual set and are thus generic. According to the *Baire category theorem* (cf. e.g. [5, Theorem 48.2]), any complete metric space is a Baire space.

Lemma 3.2. *$\mathcal{P}$ equipped with the metric*

$$d(\mu,\mu') := \sup\{|\mu(F)-\mu'(F)|;\ F\in\mathcal{F}_V\}. \tag{3.2}$$

is a complete metric space.

Proof. Let (μ_n) be a Cauchy sequence in $\mathcal{P}$ and consider

$$\mu(F) := \lim_{n\to\infty}\mu_n(F), \qquad F\in\mathcal{F}.$$

Clearly $\mu(\Omega)=1,\ \mu(\emptyset)=0$ and for any disjoint sets $F,F'\in\mathcal{F}$ we have

$$\mu(F\sqcup F') = \mu(F)+\mu(F'). \tag{3.3}$$

Suppose further that (A_m) is a sequence of elements of $\mathcal{F}$ such that $A_m\downarrow\emptyset$. Fix some $\delta>0$, and note that

- by the Cauchy property there exists some $N\geq 0$ such that

$$d(\mu_N,\mu_n) < \frac{\delta}{2}, \qquad n\geq N,$$

- by the "continuity from above" property of probability measures ([2, Theorem 2.1 (ii)]) there exists some $M\geq 0$ such that

$$\mu_N(A_m) < \frac{\delta}{2}, \qquad m\geq M.$$

[3]As Radner allows heterogeneous beliefs his $\mathcal{P}$ is more correctly identified by Δ_{N-1}^{I}

Hence

$$\mu_n(A_m) < \mu_N(A_m) + d(\mu_N, \mu_n) < \delta, \qquad m \geq M,\ n \geq N.$$

and

$$\lim_{m\to\infty} \mu(A_m) = 0. \tag{3.4}$$

Suppose that (F_k) is a sequence of disjoint elements of $\mathcal{F}_V$ and define

$$F := \bigsqcup F_k.$$

Defining

$$A_m := \bigsqcup_{k>m} F_m = F \setminus \bigsqcup_{k\leq m} F_m,$$

we have that $A_m \downarrow \emptyset$ as $m \to \infty$, and by (3.3) and (3.4)

$$\mu(F) = \sum_{k=1}^{m} \mu(F_k) + \mu(A_m) = \lim_{m\to\infty} \sum_{k=1}^{m} \mu(F_k).$$

Hence μ is a probability measure on $(\Omega, \mathcal{F}_V)$. As

$$\mu_n(F) = 0 \forall n \implies \mu(F) = 0,$$

we have that $\mu << P$, i.e. $\mu \in \mathcal{P}$. □

As indicated in the Introdution our aim is to prove that the set of confounding probability measure is meagre in the product space $\mathcal{P}^{\otimes 2} := \mathcal{P} \times \mathcal{P}$. The following lemma shows that we may, without loss of genericity, consider only probability measures that are *equivalent* to $P|_{\mathcal{F}_V}$, i.e. belonging to the set $\mathcal{P}_+$ of probability measures such that $\mu(F) > 0$ for all $F \in \mathcal{F}_V$ for which $P(F) > 0$.

Lemma 3.3. *$\mathcal{P}_+$ is a residual subset of $\mathcal{P}$ in the topology induced by the metric d.*

Proof. Clearly for any $F \in \mathcal{F}_V$ the set

$$\mathcal{P}_0(F) := \{\mu \in \mathcal{P};\ \mu(F) = 0\}$$

is closed in $\mathcal{P}$. For any $\mu \in \mathcal{P}_0(F)$ and any $\delta > 0$ there exists some $\mu' \in \mathcal{P}_0(F)^C$ such that $d(\mu, \mu') < \delta$. Hence, the interior of (the closure of) $\mathcal{P}_0(F)$ is empty and the set is nowhere dense. As $\mathcal{F}_V$ is separable, $\mathcal{P}_+^C$ is a countable union of nowhere dense sets and hence meagre. □

Remark 3.1. With the topology induced by the Lebesgue measure on $\mathbb{R}^{N-1}$, a set in $\Delta_{N-1}^{\otimes 2}$ whose closure has zero measure is clearly meagre.

4 The auxiliary proposition

For the auxiliary proposition to hold true we make the following assumptions regarding the agents' utility functions and endowments, the final payoffs and the possible equilibria:

Assumption 4.1. For every agent i,

- U_{0i}, U_i are twice continuously differentiable, strictly increasing and strictly concave, and
- $U'_{0i}(c) \to \infty$ and $U'_i(c) \to \infty$ as $c \to 0$

Assumption 4.2. Denoting $\tilde{e}^{(i)} := [\epsilon^{(i)}\ e^{(i)\top}]^\top$ we have that

- $\tilde{e}^{(i)} \in \mathbb{R}^J_+ \backslash \{0\}$ for every i and
- the sum has only strictly positive components, denoted $\sum \tilde{e}^{(i)} \in \mathbb{R}^J_{++}$.

Assumption 4.3.

1. V is bounded from above and away from zero below, in all components a.s.
2. None of the assets are redundant, i.e. there is no non-zero $x \in \mathbb{R}^J$ such that
$$V^\top x = 0 \qquad \text{a.s.}$$
3. In equilibrium there is no collection $(x^{(i)}) \in (\mathbb{R}^J)^I$ such that
$$\sum V^\top x^{(i)} U'_i(V^\top z^{(i)}) = 1 \qquad \text{a.s.}$$

Remark 4.1. These assumptions correspond roughly to the assumptions (A1)-(A3) in [6]. The assumption that $U'_i(c) \to \infty$ as $c \to 0$ is added to ensure the existence of a solution to the agents' optimisation problem in the case were short-selling is allowed. Part 3 of Assumption 4.3 is stronger in our case, but we do think that this is necessary also in the original paper. Regarding this part it may seem odd to make a priori assumptions about the properties of an equilibrium. For a justification of this point, see [6, Appendix]. Radner also assumed that the market is incomplete. In the present setting this does not seem to be necessary - it is of course the case when $\mathcal{F}$ is infinite.

The auxiliary proposition. *Under Assumptions 4.1-4.3, the set of confounding couples in $\mathcal{P}^{\otimes 2}$ is meagre.*

Before proving the auxiliary proposition we need to study the agents' demand functions. By the assumptions 4.1 and 4.3 (part 1), any equilibrium price vector has only strictly positive components, i.e. $q \in \mathbb{R}^J_{++}$ and each agent must exhaust his budget, i.e. (1.1) must hold with equality. Moreover, any agent's optimal portfolio must satisfy

$$q^\top z < \epsilon^{(i)} + q^\top e^{(i)} \text{ and } V^\top z > 0 \ \mu\text{-a.s.} \tag{4.1}$$

Given the asset price vector $q \in \mathbb{R}^J_{++}$, probability measure $\mu \in \mathcal{P}$ and portfolio $z \in \mathbb{R}^J_+$ such that (4.1) holds and $U_i'(V^\top z) \in L_1(\mathcal{F}_V, \mu)$, agent i's expected marginal utility is given by the vector

$$\Psi^{(i)}(z, q, \mu) := -U_{0i}'\big(\epsilon^{(i)} + q^\top(e^{(i)} - z)\big)q + \int_\Omega U_i'\big(V^\top z\big)V d\mu.$$

The first-order condition for (3.1) is

$$\Psi^{(i)}(z, q, \mu) = 0$$

which has a solution thanks to Assumption 4.1. If $\mu \in \mathcal{P}_+$ the matrix

$$\begin{aligned} D_z\Psi^{(i)}(z, q, \mu) &:= \begin{bmatrix} \frac{\partial \Psi^{(i)}}{\partial z_1} & \dots & \frac{\partial \Psi^{(i)}}{\partial z_J} \end{bmatrix} \\ &= U_{0i}''\big(c^{(i)}\big)qq^\top + \int_\Omega U_i''(V^\top z)VV^\top d\mu \end{aligned}$$

where $c^{(i)} := \epsilon^{(i)} + q^\top(e^{(i)} - z^{(i)})$ is negative definite because part 2 of Assumption 4.3 ensures that for any non-zero $x \in \mathbb{R}^J$

$$x^\top D_z\Psi_i(z, q)x \quad = \quad U_{i0}''\big(c^{(i)}\big)(q^\top x)^2 \ + \ \int_\Omega U_i''(V^\top z)(V^\top x)^2 d\mu \quad < \quad 0.$$

Hence, the solution $z^{(i)} = z^{(i)}(q, \mu)$ to agent i's optimisation problem is unique. To investigate its sensitivity to changes in q, consider

$$\begin{aligned} D_q\Psi^{(i)}(z^{(i)}, q, \mu) &:= \begin{bmatrix} \frac{\partial \Psi^{(i)}(z^{(i)},q,\mu)}{\partial q_1} & \dots & \frac{\partial \Psi^{(i)}(z^{(i)},q,\mu)}{\partial q_J} \end{bmatrix} \\ &= -U_{0i}'(c^{(i)})\mathbf{I} - U_{0i}''(c^{(i)})q(e^{(i)} - z^{(i)})^\top + D_z\Psi^{(i)}D_qz^{(i)}, \end{aligned}$$

where $\mathbf{I}$ is the $J \times J$ identity matrix. As $D_z\Psi^{(i)}$ is negative definite and hence nonsingular and

$$D_q\Psi^{(i)}(z^{(i)}, q, \mu) = 0$$

we have

$$D_qz^{(i)}(q, \mu) = D_z\Psi^{(i)^{-1}}\Big(U_{0i}'(c^{(i)})\mathbf{I} + U_{0i}''(c^{(i)})q(e^{(i)} - z^{(i)})^\top\Big).$$

Suppose that $\nu \in \mathcal{P}_+$ is such that $U_i'(V^\top z^{(i)}) \in L_1(\mathcal{F}_V, \nu)$ as well. Then we can define the *directional derivative*

$$\begin{aligned} D_{\mu,\nu}\Psi^{(i)}(z^{(i)}, q, \mu) := \lim_{\delta \to 0} \frac{1}{\delta}\Big(&\Psi^{(i)}\big(z^{(i)}(q, (1-\delta)\mu + \delta\nu), q, (1-\delta)\mu + \delta\nu\big) \\ &-\Psi^{(i)}\big(z^{(i)}(q,\mu), q, \mu\big)\Big) \\ =\ & D_z\Psi^{(i)} D_{\mu,\nu} z^{(i)} + \int_\Omega U_i'(V^\top z^{(i)}) V(d\nu - d\mu). \end{aligned}$$

Reasoning as above we have that

$$D_{\mu,\nu} z^{(i)}(q,\mu) = D_z\Psi^{(i)^{-1}} \int_\Omega U_i'(V^\top z^{(i)}) V(d\mu - d\nu).$$

Remark 4.2. In Radner's paper, short selling is not allowed and the first-order condition for (3.1) is

$$\begin{aligned} \Psi_j^{(i)}(z^{(i)}, q, \mu) &= 0 \qquad && \text{if } z_j^{(i)} > 0 \\ \Psi_j^{(i)}(z^{(i)}, q, \mu) &\le 0 \qquad && \text{if } z_j^{(i)} = 0 \end{aligned}$$

The demand $z_j^{(i)}$ can fail to be differentiable in q and/or μ only if

$$\Psi_j^{(i)}(z^{(i)}, q, \mu) \text{ and } z_j^{(i)} = 0.$$

It is proved that this can only happen in equilibrium for a negligible set of probability measures. In the infinite dimensional setting, we can prove that this will only happen for a meagre set of probability measures.

Lemma 4.1. *For any $q \in \mathbb{R}^J_{++}$ the set $\mathcal{P}(q)$ of probability measures for which q is an equilibrium price is a meagre subset of $\mathcal{P}$.*

Proof. Suppose that $(\mathcal{P}_n)$ is an increasing sequence of closed sets in $\mathcal{P}_+$ such that[4]

$$\mathcal{P}_+ = \bigcup_{n=1}^\infty \mathcal{P}_n.$$

By the continuity properties of the $z^{(i)}$'s, the set

$$\mathcal{P}_n(q) := \Big\{\mu \in \mathcal{P}_n;\ \sum z^{(i)}(q,\mu) = \sum e^{(i)}\Big\}$$

is closed. Suppose that $\mathcal{P}_n(q)$ has non-empty interior: then there exists some open set B in $\mathcal{P}_n(q)$ such that for any μ, ν in B

$$\sum D_{\mu,\nu} z^{(i)}(q,\mu) = \sum D_z\Psi^{(i)^{-1}} \int_\Omega U_i'(V^\top z^{(i)}) V s(d\nu - d\mu) = 0,$$

[4]For given $\nu \in \mathcal{P}_+$, $\mathcal{P}_n$ could be the set of μ's in $\mathcal{P}$ such that $\mu(F) \geq \frac{1}{n}\nu(F)$ for all $F \in \mathcal{F}_V$

which implies that there exists some $y \in \mathbb{R}^J$ such that

$$\sum D_z \Psi^{(i)^{-1}} U_i'(V^\top z^{(i)})V = y \qquad \text{a.s.}$$

By part 2 of Assumption 4.3 any such y can have only non-zero components, which in turn implies that part 3 is violated. Hence $\mathcal{P}_n(q)$ has empty interior and

$$\mathcal{P}(q) \subseteq \bigcup_{n=1}^{\infty} \mathcal{P}_n(q) \bigcup \mathcal{P}_+^C$$

is meagre. □

Lemma 4.2. *For any $\mu \in \mathcal{P}_+$, there is a countable number of equilibrium prices.*

Proof. It is sufficient to prove that the set of equilibrium prices is closed and that any perturbation of the asset prices will bring the economy out of equilibrium, i.e.

$$x^\top D_q \sum z^{(i)}(q, \mu) = 0 \Leftrightarrow x = 0. \tag{4.2}$$

The continuity of the agents' demands as functions of q ensures that the set of equilibrium prices is closed. As any equilibrium corresponds to a no-trade equilibrium in the case where all agents have the endowment $e^{(i)} = z^{(i)}(q, \mu)$ we may, without loss of generality assume this and consider

$$\sum D_q z^{(i)}(q, \mu) = \sum D_z \Psi^{(i)^{-1}} U_{0i}'(c^{(i)}).$$

But this is a sum of positive definite matrices and hence non-singular and (4.2) holds. □

Proof of the auxiliary proposition. Let $\mathcal{Q}_n \subseteq \mathcal{P}^{\otimes 2}$ denote the set of confounding couples each belonging to $\mathcal{P}_n$ and such that their common equilibrium price is bounded componentwise by $\frac{1}{n}$ from below and n from above. The set of confounding couples is clearly contained in the union of $\bigcup \mathcal{Q}_n$ and ${\mathcal{P}_+^{\otimes 2}}^C$. As the latter is clearly meagre it is sufficient to show that all the $\mathcal{Q}_n$'s are meagre. Suppose (μ_m, ν_m) is a sequence in $\mathcal{Q}_n$ converging to (μ, ν). The boundedness of the common equilibrium prices ensures that there is some subsequence of these prices that converge to some $q \in \mathbb{R}^J$ whose components are in $[\frac{1}{n}, n]$. The continuity of the agent's demands near an equilibrium ensures that q is a common equilibrium price for (μ, ν). The couple must then belong to $\mathcal{Q}_n$, which is closed. By the Lemmas 4.1 and 4.2, in any vicinity of ν there is some ν' such that (μ, ν') is *not* confounding. Hence, the interior of (the closure of) $\mathcal{Q}_n$ is empty and the set is nowhere dense. □

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